**Solving simultaneous equations graphically**

 **A LEVEL LINKS**

 **Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

* You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples

**Example 1** Solve the simultaneous equations *y* = 5*x* + 2 and *x* + *y* = 5 graphically.

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| *y* = 5 – *x**y* = 5 – *x* has gradient –1 and *y*-intercept 5.*y* = 5*x* + 2 has gradient 5 and *y*-intercept 2.Lines intersect at *x* = 0.5, *y* = 4.5Check:First equation *y* = 5*x* + 2: 4.5 = 5 × 0.5 + 2 YESSecond equation *x* + *y* = 5: 0.5 + 4.5 = 5 YES | **1** Rearrange the equation *x* + *y* = 5 to make *y* the subject.**2** Plot both graphs on the same grid using the gradients and *y*-intercepts.**3** The solutions of the simultaneous equations are the point of intersection.**4** Check your solutions by substituting the values into both equations. |

**Example 2** Solve the simultaneous equations *y* = *x* − 4 and *y* = *x*2 − 4*x* + 2 graphically.

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| --- | --- | --- | --- | --- | --- |
| ***x*** | 0 | 1 | 2 | 3 | 4 |
| ***y*** | 2 | –1 | –2 | –1 | 2 |

The line and curve intersect at *x* = 3, *y* = −1 and *x* = 2, *y* = −2Check:First equation *y* = *x* − 4: −1 = 3 − 4 YES −2 = 2 − 4 YESSecond equation *y* = *x*2 − 4*x* + 2: −1 = 32 − 4 × 3 + 2 YES −2 = 22 − 4 × 2 + 2 YES | **1** Construct a table of values and calculate the points for the quadratic equation.**2** Plot the graph.**3** Plot the linear graph on the same grid using the gradient and *y*-intercept.*y* = *x* – 4 has gradient 1 and *y*-intercept –4.**4** The solutions of the simultaneous equations are the points of intersection.**5** Check your solutions by substituting the values into both equations. |

Practice

**1** Solve these pairs of simultaneous equations graphically.

 **a** *y* = 3*x* − 1 and *y* = *x* + 3

 **b** *y* = *x* − 5 and *y* = 7 − 5*x*

 **c** *y* = 3*x* + 4 and *y* = 2 − *x*

**2** Solve these pairs of simultaneous equations graphically.

**Hint**

Rearrange the equation to make *y* the subject.

 **a** *x* + *y* = 0 and *y* = 2*x* + 6

 **b** 4*x* + 2*y* = 3 and *y* = 3*x* − 1

 **c** 2*x* + *y* + 4 = 0 and 2*y* = 3*x* − 1

**3** Solve these pairs of simultaneous equations graphically.

 **a** *y* = *x* − 1 and *y* = *x*2 − 4*x* + 3

 **b** *y* = 1 − 3*x* and *y* = *x*2 − 3*x* − 3

 **c** *y* = 3 − *x* and *y* = *x*2 + 2*x* + 5

**4** Solve the simultaneous equations *x* + *y* = 1 and *x*2 + *y*2 = 25 graphically.

Extend

**5** **a** Solve the simultaneous equations 2*x* + *y* = 3 and *x*2 + *y* = 4

 **i** graphically

 **ii** algebraically to 2 decimal places.

 **b** Which method gives the more accurate solutions? Explain your answer.

Answers

**1 a** *x* = 2, *y* = 5

 **b** *x* = 2, *y* = −3

 **c** *x* = −0.5, *y* = 2.5

**2 a** *x* = −2, *y* = 2

 **b** *x* = 0.5, *y* = 0.5

 **c** *x* = −1, *y* = −2

**3 a** *x* = 1, *y* = 0 and *x* = 4, *y* = 3

 **b** *x* = −2, *y* = 7 and *x* = 2, *y* = −5

 **c** *x* = −2, *y* = 5 and *x* = −1, *y* = 4

**4** *x* = −3, *y* = 4 and *x* = 4, *y* = −3

**5 a i** *x* = 2.5, *y* = −2 and *x* = −0.5, *y* = 4

 **ii** *x* = 2.41, *y* = −1.83 and *x* = −0.41, *y* = 3.83

 **b** Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.