Problem Sheet 1 – Number I

Question 1	$27 \times 147 = (3 \times 3 \times 3) \times (3 \times 7 \times 7)$ $= (3 \times 3 \times 7) \times (3 \times 3 \times 7)$ $= 63 \times 63$. So the answer is 63.
Question 2	If the average of three numbers is 8, their total must be 24. Since two of the numbers are 5 and 13, and $5 + 13 = 18$, the other number must be 6.
Question 3	$\frac{101!}{99!} = \frac{101 \times 100 \times 99 \times 98 \times \ldots \times 4 \times 3 \times 2 \times 1}{99 \times 98 \times 97 \times \ldots \times 4 \times 3 \times 2 \times 1}$ $= 101 \times 100 = 10100$.
Question 4	Expressing the numbers as products of primes: $2 = 2$, $3 = 3$, $4 = 2 \times 2$, $5 = 5$, $6 = 2 \times 3$, $7 = 7$, $8 = 2 \times 2 \times 2$, $9 = 3 \times 3$. The number needed should include all the primes listed above. The number of times each prime is included is given by the maximum number of times it appears in the list above. So the number is $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$.

Question 5

$$\frac{(101^4-4)(101^4-1)}{(101^2-2)(101^2-1)} - \frac{(101^4-4)(101^4-1)}{(101^2-2)(101^2+1)}$$

$$= \frac{(101^2+2)(101^2-2)(101^2+1)(101^2-1)}{(101^2-2)(101^2-1)}$$

$$- \frac{(101^2+2)(101^2-2)(101^2+1)(101^2-1)}{(101^2-2)(101^2+1)}$$

$$= (101^2+2)\left[101^2+1-(101^2-1)\right]$$

$$= (101^2+2) \times 2$$

$$= 20406.$$

Question 6

$$\frac{(10!+9!)(8!+7!)(6!+5!)(4!+3!)(2!+1!)}{(10!-9!)(8!-7!)(6!-5!)(4!-3!)(2!-1!)}$$

$$= \frac{9!(10+1)\,7!(8+1)\,5!(6+1)\,3!(4+1)\,1!(2+1)}{9!(10-1)\,7!(8-1)\,5!(6-1)\,3!(4-1)\,1!(2-1)}$$

$$= \frac{11 \times 9 \times 7 \times 5 \times 3}{9 \times 7 \times 5 \times 3 \times 1} = 11.$$

Question 7

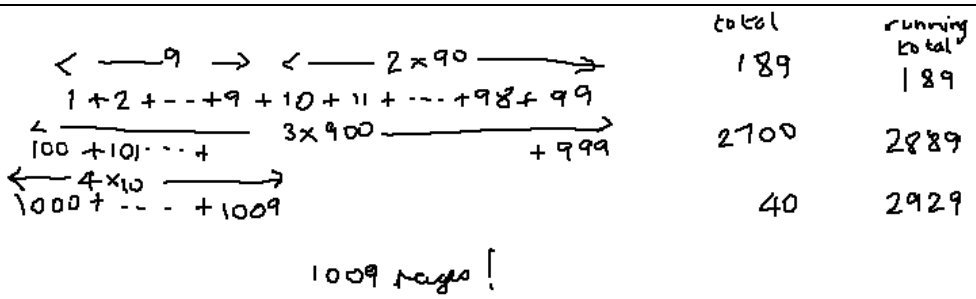
Since the mean is 20, the five numbers must total to 100. If the numbers are arranged in ascending order, the middle number is 21. There are at least two 24s. If there were more than two 24s the middle number would be 24. So there must be two 24s.

The list has the form

$$x, y, 21, 24, 24$$

where $x + y = 31$ and $x, y \leq 21$

Question 8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189
1+2+--+9+10+11+---+98+99
100+101---+
3×900
+999
2700
2889
4×10
1000+---+1009
40
2929
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